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PLENARY TALKS

BERNSTEIN-SCHNABL OPERATORS, APPROXIMATION PROBLEMS AND INITIAL - LATERAL VALUE PROBLEMS FOR DIFFUSION EQUATIONS ON [0,1]

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2000 Mathematics Subject Classification. 41A36, 47D07, 35K20

Keywords and phrases. Positive operator, Bernstein-Schnabl operator, Markov semigroup, initial-lateral value problem for diffusion equations

The lecture will report some of the main results which first appeared in the papers [1] and [2] and which are concerned with diverse types of approximation problems in the space $C([0,1])$.

More recently, these results have been notably deepened and generalized to spaces of the form $C(K)$, K being a (not necessarily finite-dimensional) convex compact subset, originating a comprehensive series of results (a theory) which is documented in the forthcoming monograph [3].

However, for the sake of simplicity, the discussion will be limited to the unit interval where, among other things, more complete results can be shown.

The lecture will focus on a sequence of positive linear operators on $C([0,1])$, which are referred to as Bernstein-Schnabl operators.

These operators, which generalize the classical Bernstein operators, not only furnish new general approximation processes for continuous functions on $[0,1]$ but they also approximate the Markov semigroups which govern initial-lateral value problems for (degenerate) diffusion equations of the form

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = \alpha(x)\frac{\partial^2 u(x,t)}{\partial x^2} & 0 < x < 1, \ t \geq 0, \\ \lim\limits_{x\to 0^+} \alpha(x)\frac{\partial^2 u(x,t)}{\partial x^2} = \lim\limits_{x\to 1^-} \alpha(x)\frac{\partial^2 u(x,t)}{\partial x^2} = 0 & t \geq 0, \\ \lim\limits_{t\to 0^+} u(x,t) = u_0(x) & 0 \leq x \leq 1, \end{cases} \tag{1}$$

where the function α is continuous and positive on $[0,1]$ and $0 < \alpha(x) \leq \frac{x(1-x)}{2} \quad (0 < x < 1)$.

Problems as in (1) occur in some models from population genetics.

Both approximation and shape preserving properties of Bernstein-Schnabl operators will be discussed together with their counterparts for the relevant Markov semigroups which, in turn, lead to spatial regularity properties to the solutions $u(\cdot,t)$, $t \geq 0$, of problems (1). In particular, the asymptotic behaviour

$$\lim_{t\to+\infty} u(x,t) \quad \text{uniformly with respect to } x \in [0,1]$$

is determined as well.

Finally, the saturation class of Bernstein-Schnabl operators and the Favard class of the corresponding Markov semigroup are determined.

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ADAPTIVE VOLUME BASED ALGORITHM FOR POLYHEDRAL APPROXIMATION OF 3D SOLIDS

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2000 Mathematics Subject Classification. 41A35, 41A63

Keywords and phrases. Solid mesh; spatial decomposition; polyhedral approximation.

In this talk we give an adaptive method for the approximation of three-dimensional solids. We construct an iterative method that can be used for approximation of regular subsets of $\mathbb{R}^3$. We will discuss the connection between solid meshes, regular sets and polyhedra. First the general description of the method will be given. The main idea of our algorithm is a kind of spatial decomposition with increasing atomic σ-algebra sequences. In every step one atom will be divided into two nonempty atoms. We define a volume-based distance metric, and we give strategies for choosing and dividing the atoms and compute the boundary of the approximating solid. As we use planes for dividing the atoms, the newly created elements and its finite union are polyhedra. Details of our approximation will be demonstrated on triangular meshes, we will suggest a data structure for polyhedra to compute the intersection, union and boundary of the atoms fast and easily.

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CONSTRUCTION AND APPLICATIONS OF GAUSSIAN QUADRATURES WITH NONSTANDARD AND EXOTIC WEIGHT FUNCTIONS

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Keywords and phrases. Orthogonal polynomials; moments; recursion coefficients; Gaussian quadtarure; weight function.

Let w be a given non-negative weight function od (a, b) for which all moments $\mu_k = \int_a^b x^k w(x)\mathrm{d}x$, $k = 0, 1, \ldots$, exist and $\mu_0 > 0$. Then the sequence of (monic) orthogonal polynomials $\{\pi_k\}_{k=0}^{+\infty}$ exists and such polynomials satisfy the three-term recurrence relation $\pi_{k+1}(x) = (x - \alpha_k)\pi_k(x) - \beta_k \pi_{k-1}(x)$, $k = 0, 1, \ldots$, with starting values $\pi_0(x) = 1$, $\pi_{-1}(x) = 0$. The recursive coefficients $\alpha_k = \alpha_k(w)$ and $\beta_k = \beta_k(w)$ depend on a given weight function and they are known in an explicit form only for a narrow class of weight functions, e.g. for classical weights, and some others similar to them. For others, the so-called "strong non-classical weights", a numerical construction of recursive coefficients can be a very dificult problem, because the map of moments into these coefficients is ill-conditioned (cf. [2] and [4]).

The corresponding Gaussian quadratures

$$\int_a^b f(x)w(x)\mathrm{d}x = \sum_{\nu=1}^{n} A_\nu^{(n)} f(x_\nu^{(n)}) + R_n(f)$$

exist uniquely for each $n \in \mathbb{N}$ and $R_n(f) \equiv 0$ when f is a polynomial of degree at most $2n-1$. Knowing the recursive coefficients α_k and β_k for $k \le N-1$, the nodes $x_\nu^{(n)}$ and the Christoffel numbers $A_\nu^{(n)}$, $\nu = 1, \dots, n$, can be determined for each $n \le N$, for example, by the well-known Golub-Welsch algorithm [3].

Recent progress in variable precision arithmetic and symbolic computation now makes it possible to generate the coefficients α_k and β_k directly using the original Chebyshev method in sufficiently high precision (for such a software in MATHEMATICA, which is downloadable from Web Site: `http://www.mi.sanu.ac.rs/~gvm/` see [1] and [5]).

In this lecture we present the construction of several quadratures of Gaussian type with respect to strong non-classical weights and exotic weight functions which appear in approximation theory, summation of slowly convergent series, statistics, fraction calculus, etc. Also, we give some applications of such kind of quadratures.

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POSITIVE LINEAR POLYNOMIAL OPERATORS AND THEIR EIGENVALUES

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Keywords and phrases. Linear polynomial operators; positive operators; Bernstein-type polynomials; eigenvalues.

We will consider linear polynomial operators that use certain information of the functions (for instance, values of the functions in some points, preserving one, or several), shape preserving properties (positivity, monotonicity, convexity, ...) and with good "behaviour" for polynomial of low degree.

The space of polynomials that can be reproduced for shape preserving linear operators is related with Korovkin-type theorems and the maximum value of the eigenvalues is related with Berens-DeVore-type results. The maximum value of the eigenvalue is related with the error estimation.

The Bernstein operator is the classical example of positive linear polynomial operator but its eigenvalues could be improved if we consider only some preserving properties.

We will consider different data and shape properties.

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MULTISCALE ANALYSIS AND SIMULATION OF TRANSPORT PROCESSES THROUGH MEMBRANES

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Keywords and phrases. Nonlinear reaction-diffusion systems; thin heterogeneous layer; transmission conditions; homogenization; two-scale convergence; numerical multiscale analysis.

In our contribution, we develop multiscale methods for the derivation and analysis of effective models in environments containing membranes. At the microscopic level, where membranes are modeled as thin heterogeneous layers, the model consists of reaction-diffusion equations within each subdomain. At the macroscopic level, membranes are reduced to interfaces, and effective transmission conditions at these interfaces are formulated. These transmission conditions involve averages over fluxes corresponding to solutions of so called cell-problems. The derived models can be applied e.g. in the mathematical modeling of intracellular processes, where intracellular membrane systems essentially influence the spatial distribution of cellular compounds.

Our asymptotic analysis is based on weak and strong two-scale convergence results for sequences of functions defined on thin heterogeneous layers. For the derivation of the effective transmission conditions, we develop a new method based on test functions of boundary-layer type, see [1]. In [3], the techniques developed in [1] for the derivation of effective transmission conditions across flat interfaces are further extended to include curved membranes. This step requires new concepts like periodicity on manifolds and curved layers, and two-scale convergence with respect to charts.

For the analysis of the effective (homogenized) problem an appropriate function space, which includes the coupling conditions for the concentrations on the interface, is introduced. The transmission conditions for the flux are included in the variational formulation with respect to this function space. The solution of the effective problem is approximated by using the Galerkin method. Numerically the problem is reduced to a system of ordinary differential equations, where the coupling of the micro- and macro- variables leads to a special structure, distinguishing the variables relevant for the transmission. Results of numerical simulations are illustrating the effect of the microscopic process in the membrane on the macroscopic reaction-diffusion process in the bulk domains, see [2]. Parts of this contribution are obtained jointly with Willi Jäger and Stephan Ludwig, University of Heidelberg.

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NUMERICAL SCHEMES FOR TWO-PHASE POROUS MEDIA FLOW WITH DYNAMIC CAPILLARITY EFFECTS

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Keywords and phrases. Pseudo-parabolic equations; weak solutions; discretization; two-phase flow in porous media; dynamic capillarity.

We consider a mathematical model for two-phase flow in porous media. The particular aspect is in including dynamic terms in the capillary pressure [6]. Compared to traditional models, the resulting model is pseudo-parabolic and includes a third order mixed derivatives term modeling the dynamic effects in the capillary pressure:

$$\partial_t u + \partial_x f(u) = \varepsilon \partial_x \left(H(u) \partial_x (u + \varepsilon \tau \partial_t u) \right).$$

Typically, f is a convex-concave shaped flux function and H is a model related non-negative nonlinearity that might degenerate. Furthermore, $\varepsilon > 0$ is related to the capillary number, the ratio between capillary and viscous forces, and $\tau \geq 0$ quantifies the influence of the dynamic effects in the capillary pressure.

For the model above we first investigate the existence of weak solutions (see [5, 7, 8]). After discussing some feature of these solutions, including the possibility to have non-monotonic profiles [3], we consider different numerical formulations [2, 4]. Finally, we provide numerical results in both homogeneous and heterogeneous media, and compare these to the experiments in [1].

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RANDOM DYNAMICAL SYSTEMS FOR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS DRIVEN BY FRACTIONAL BROWNIAN MOTIONS

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2000 Mathematics Subject Classification. 60H15

Keywords and phrases. Stochastic partial differential equations

In this talk we are concerned with the study of the existence and uniqueness of pathwise mild solutions to stochastic evolutions equations driven by a Hölder continuous function with Hölder exponent H in $[1/2, 1)$ and $(1/3, 1/2)$ and with non-linear coefficients in front of the noise. To be more precise, for $H \in (1/3, 1/2)$ an stochastic integral is defined by using a fractional integration by parts formula and it involves a tensor-valued component for which we formulate a special equation. From this it turns out that we have to solve a system consisting in a path and an area equation. We will address existence of a unique pathwise solution of this system of equations. From this pathwise property we can derive the existence of a random dynamical system.

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CONTRIBUTED TALKS

A SECOND-ORDER FINITE DIFFERENCE SCHEME FOR THE CAHN-HILLIARD EQUATION

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2000 Mathematics Subject Classification. 65N06, 65N12, 35K55, 35K30

Keywords and phrases. Cahn–Hilliard equation; second-order finite difference method; Thomas algorithm; conservation of energy; error estimate.

A second-order finite difference method is analyzed for the approximation of the solution of the evolutionary fourth–order in space Cahn-Hilliard equation. This equation is modeled with inertial term in order to model spinodal decomposition caused by deep supercooling in certain glasses. We prove that the scheme has a unique solution and we study error estimation for the numerical scheme.

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WEIGHTED OSTROWSKI-GRÜSS-TYPE INEQUALITIES

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Keywords and phrases. Ostrowki-Grüss type inequalities; modulus of smoothness; least concave majorant of modulus of continuity.

Several inequalities of Ostrowski-Grüss-type availabe in the literature are generalized considering the weighted case of them. Involving the least concave majorant of the modulus of continuity we provide the upper bounds of our inequalities. This is joint work with Heiner Gonska (University of Duisburg-Essen, Germany).

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SOME APPROXIMATION PROPERTIES OF CERTAIN NONLINEAR BERNSTEIN OPERATORS

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2000 Mathematics Subject Classification. 41A35, 41A25, 47G10

Keywords and phrases. Nonlinear Bernstein operators; modulus of continuity; Voronovskaya-type formula; pointwise convergence.

In this work we deal with the pointwise approximation properties of a certain sequence of nonlinear Bernstein operators. First of all, we will give existence theorem for these operators on Lebesgue spaces, then we will investigate its rate of convergence by using the modulus of continuity. Furthermore, we present a Voronovskaya type formula for these operators.

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DISCRETE OPERATORS ASSOCIATED WITH THE DURRMEYER OPERATOR

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2000 Mathematics Subject Classification. 41A36

Keywords and phrases. Positive linear operators; quadrature.

In [1] the author constructed discrete operators associated with certain integral operators using a probabilistic approach. In this talk we obtain positive linear operators of discrete type associated with the classical Durrmeyer operator with the aid of some quadrature formulas with positive coefficients. Using Gaussian quadratures we get operators which preserves the moments of the classical Durrmeyer operator up to a given order. Another class of discrete operators is obtained by using the quadratures generated by some positive linear operators. We study the convergence of the new operators and compare them with the Durrmeyer operator. Also, we present some problems of optimality and give numerical examples.

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DYNAMICAL SYSTEMS ON FRACTAL SETS

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2000 Mathematics Subject Classification. 37H10, 28A80, 37L05

Keywords and phrases. Analytic semigroups; p.c.f. fractals; stochastic differential equations; random dynamical systems.

We will consider random dynamical systems in a special function space on a class of fractals.
First we summarize important notations about the fractal setting and its function spaces we are interested in. Further we introduce analytic semigroups in order to define mild solutions of stochastic differential equations on fractal sets. Finally a solution of this stochastic differential equation generates a random dynamical system.

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A SUCCESSIVE APPROXIMATION TECHNIQUE FOR THE SOLUTION OF A BILOCAL PROBLEM

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2000 Mathematics Subject Classification. 35A35, 47G10, 47H10, 54H25

Keywords and phrases. Bilocal problem; Contraction Principle; Fredholm integral operator; successive approximation sequence.

Using a method proposed by T.A. Burton in [3] and the Contraction Principle, we study the existence, uniqueness and approximation of the solution for a bilocal problem. We compare our results with the classical similarly results given for the Fredholm integral operator associated.

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DIFFERENTIAL OPERATORS, MARKOV SEMIGROUPS AND BERNSTEIN-SCHNABL OPERATORS

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2000 Mathematics Subject Classification. 47F05, 41A36, 41A63.

Keywords and phrases. Positive approximation process; elliptic-second order differential operator; Markov semigroup; approximation of semigroups.

The results presented in this talk are contained in [1] and in the monograph [2].

Let K be a convex compact subset of $\mathbf{R}^d$, $d \geq 1$, and denote by $C(K)$ the space of all real-valued continuous functions on K. Given a Markov operator T on $C(K)$, in association with T we introduce and study a suitable positive approximation process on $C(K)$, namely the sequence $(B_n)_{n\geq 1}$ of the so-called Bernstein-Schnabl operators associated with T.

We also introduce a class of second-order elliptic differential operators on K and we prove that they are related with the B_n's by means of an asymptotic formula.

As a matter of fact, the closures of these differential operators generate Markov semigroups on $C(K)$. Moreover, those semigroups may be approximated by iterates of the Bernstein-Schnabl operators.

Among other things, this allow us to infer some spatial regularity properties of the solutions to the diffusion problems governed by the above mentioned differential operators by deriving them from the relevant preserving properties held by the operators B_n.

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ON THE CONVERGENCE OF AN AITKEN-NEWTON TYPE METHOD

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2000 Mathematics Subject Classification. 65H05

Keywords and phrases. Nonlinear equations in $\mathbb{R}$; iterative methods; local convergence.

We study the solving of nonlinear equations in $\mathbb{R}$ by an iterative method of Aitken-Newton type, which uses the Hermite inverse interpolatory polynomial of degree 2. We obtain a local convergence result which shows that the q-convergence order of this method is 8, and the efficiency index is $\sqrt[5]{8}$.

Under certain assumptions (including monotonicity and convexity of the nonlinear mapping) this method is shown that generates monotone sequences.

We present some numerical examples which illustrate the theoretical results.

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STATISTICAL ASPECTS ON THE USAGE OF SOME DERMATOLOGICAL CREAMS WITH METALLIC NANOPARTICLES

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2000 Mathematics Subject Classification. 62P10

Keywords and phrases. Statistics; metallic nanomaterials; medical effects.

The main purpose of the present talk is to emphasize, by statistical studies, the anti-inflammatory effect of some new prepared nanomaterials on skin diseases (psoriasis). These new materials are based on silver and gold nanoparticles and natural compounds extracted from native plants of Adoxaceae family (European cranberry bush - Viburnum opulus L., European black Elderberry - Sambucus nigra L. and Cornus mas), which grow in our country and possess a known anti-inflammatory activity mainly due to their high content of anthocyanins and other polyphenols.

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THE GENERALIZATION OF MASTROIANNI OPERATORS USING THE DURRMEYER'S METHOD

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2000 Mathematics Subject Classification. 41A36, 41A25

Keywords and phrases. Mastroianni operator; operator of Durrmeyer type; approximation properties.

In the present talk, we define and study the approximation properties of a Durrmeyer's type operator associated with Mastroianni operator.

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A DIRECT APPROACH FOR PROVING WALLIS RATIO ESTIMATES AND AN IMPROVEMENT OF ZHANG-XU-SITU INEQUALITY

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Keywords and phrases. Gamma function; Wallis ratio; asymptotic series; inequalities

In time, inequalities about Wallis ratio and related functions were presented by many mathematicians using various methods such as mean inequality, Jensen inequality, monotonicity of some sequences and montonicity or complete monotonicity of some functions. The main aim of this work is to show that the natural approach for solving these inequalities is to consider and to exploit the inequalities obtained by truncation of some asymptotic series. Such inequalities provide estimates of any accuracy n^{-k}, as n approaches infinity. Finally, an improvement of an inequality due to X.-M. Zhang, T.-Q. Xu and L.-B. Situ [Geometric convexity of a function involving gamma function and application to inequality theory, *J. Inequal. Pure Appl. Math.* 8 (1) (2007) Art. 17, 9 pp.] is presented.

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ONE NEW HYBRID CONJUGATE GRADIENT METHOD AS A CONVEX COMBINATION OF FR AND PRP CONJUGATE GRADIENT METHODS

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Keywords and phrases. Hybrid conjugate gradient method; unconstrained optimization.

Another hybrid conjugate gradient algorithm for unconstrained optimization is proposed. The parameter β_k^{hyb} is computed as a convex combination of β_k^{FR} – corresponding to Fletcher and Reeves, and β_k^{PRP} – corresponding to Polak-Ribiére-Polyak conjugate gradient algorithms, i.e. $\beta_k^{hyb} = (1-\theta_k)\beta_k^{PRP} + \theta_k\beta_k^{FR}$. The parameter θ_k is computed in such a way that the conjugacy condition holds. The algorithm uses the Wolfe line search conditions. Computational results for a set of functions from [1], show that this new conjugate gradient algorithm behaves better than some known conjugate gradient algorithms.

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HIGHER ORDER ROOTS ESTIMATES OF RATIO OF GAMMA FUNCTIONS

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2000 Mathematics Subject Classification. 26D15, 11Y25, 41A25, 34E05

Keywords and phrases. Gamma function; approximations; asymptotic series

The aim of this talk is to give a systematically way for obtaining higher order roots estimates of the ratio $\frac{\Gamma(x+1)}{\Gamma\left(x+\frac{1}{2}\right)}$, as $x \to \infty$ and the Wallis ratio $\frac{1\cdot 3\cdots(2n-1)}{2\cdot 4\cdots(2n)}$, as $n \to \infty$.

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CONVERGENCE RESULT FOR A VARIATIONAL INEQUALITY

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2000 Mathematics Subject Classification. 74M15, 45D05, 46E30

Keywords and phrases. Variational inequality; Fréchet spaces; weak solution; integral equation

We prove a convergence result for a system coupling two integral equations with a history-dependent variational inequality. More exactly, we consider the variational formulation of a quasistatic contact problem with adhesion. Then we prove the dependence of the weak solution with respect to the data. The proof is based on arguments of variational inequalities, Fréchet spaces and Gronwall inequalities.

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MATRIX-PENCIL METHOD FOR ESTIMATING PARAMETERS OF MONOMIAL-EXPONENTIAL SUMS

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2000 Mathematics Subject Classification. 41A46, 15A22, 65F15

Keywords and phrases. Nonlinear approximation; parameter estimation; matrix pencil method.

In this talk we describe a numerical procedure to solve the following non-linear approximation problem: recover the positive integers M and $\{m_j\}_{j=1}^{M}$, the distinct complex or real parameters $\{f_j\}_{j=1}^{M}$ and the complex or real coefficients $\{c_{js}\}_{j=1,\,s=0}^{M,\,m_j-1}$ of the following monomial-exponential sum

$$h(x) = \sum_{j=1}^{M} \sum_{s=0}^{m_j-1} c_{js} x^s e^{f_j x}, \qquad (1)$$

knowing $h(k)$ for $2N$ equidistant data points $k = k_0, k_0+1, \ldots, k_0+2N,\ k_0 \in \mathbb{N}$ with $N \geq L = m_1 + \ldots + m_M$. The method we use is based on theoretical results proved in [1] where different algorithms are also proposed. We also show how the method can be generalized to recover parameters in the bi-variate sum

$$h(x_1, x_2) = \sum_{j=1}^{M} \sum_{s_1=0}^{m_{1j}-1} \sum_{s_2=0}^{m_{2j}-1} c_{js_1}\, c_{js_2}\, x_1^{s_1}\, x_2^{s_2}\, e^{f_{1j}x_1 + f_{2j}x_2}. \qquad (2)$$

Numerical results will also be given in order to highlight the effectiveness of the method.

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ON THE STOCHASTIC SHELL MODEL DRIVEN BY A MULTIPLICATIVE FRACTIONAL BROWNIAN MOTION

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2000 Mathematics Subject Classification. 60H15, 60G22

Keywords and phrases. Stochastic PDEs; fractional Brownian motion; pathwise solutions; fractional calculus.

We consider some shell models of turbulence in a very general form. These are phenomenological approximations of the Navier-Stokes equations, with a viscous linear part that is dissipative and a nonlinear part that is not globally Lipschitz. We assume that this model is driven by a multiplicative fractional Brownian noise with Hurst parameter $H \in (1/2, 1)$, and that the nontrivial diffusion term is a nonlinear operator satisfying some Lipschitz property and some other differentiability conditions. We will prove the existence and uniqueness of a pathwise mild solution, and the proof will be achieved in two steps. In the first step we shall prove the existence and uniqueness of variational solutions to the Shell model but driven by smooth paths, for which we are able to get some important uniform estimates in appropriate functional spaces. In a second step, by using these estimates and a compactness argument, we are able to pass to the limit, showing that this limit is a pathwise mild solution for the Shell model driven by a fractional Brownian motion.

A STOCHASTIC NEWTON-RAPHSON METHOD FOR SOLVING NONLINEAR EQUATIONS

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2000 Mathematics Subject Classification. 26D10, 46N30

Keywords and phrases. Newton's method; Monte Carlo simulations; predictor-corrector methods.

We present a simple modification of the Newton‘s method for solving nonlinear scalar equations. The family of methods obtained through this modification, maintains the second order of convergence. The proposed method originates in the recent work of Trevor J. McDougall and Simon J. Wotherspoon, [1]. The method can be used in a Monte-Carlo type enviroment which results in convergence for cases where the standard Newton methods fail to converge.

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ON BULLEN'S AND RELATED INEQUALITIES

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2000 Mathematics Subject Classification. 26D15, 26A15, 39B62

Keywords and phrases. Bullen's inequality; K-functional; modulus of smoothness; least concave majorant of modulus of continuity

The estimate in Bullen's inequality is extended for continuous functions using the second order modulus of smoothness. A different form of this inequality is given in terms of the least concave majorant. The composite case of Bullen's inequality is considered as well. This is joint work with Ana Acu (Sibiu).

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A LINEARIZATION METHOD OF A NONLINEAR STOCHASTIC SCHRÖDINGER EQUATION

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2000 Mathematics Subject Classification. 60H15

Keywords and phrases. Stochastic Schrödinger equation; approximation

We consider a one dimensional stochastic Schrödinger equation on a bounded domain with homogeneous Neumann boundaries conditions and Lipschitz nonlinearities. The equation contains a multiplicative noise which is defined by a stochastic integral with respect to a cylindrical Wiener process. The solution is defined in the weak generalized sense [GrLi]. We introduce a sequence of linearized problems with additive noise so that the solutions of these problems approximate the solution of the orginal problem.

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IMAGE OF CIRCLES THROUGH REMARCABLE ANALYTICAL FUNCTIONS

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2000 Mathematics Subject Classification. 30C45

Keywords and phrases. Special classes of univalent functions; starlike; convex; alpha-convex; spiral; geometric images of circles through remarkable functions.

As presented in [1], classical results on univalent functions are the beginning of a beautiful journey in the development of Geometric Functions Theory.

Based on that, we have studied some analytical and remarkable functions and their affiliation to some special classes of univalent functions.

Besides that, we have presented some results regarding the geometric image of circles through the above mentioned functions. The main idea that comes out here, is that from the graphical representations we can observe some properties, that a function owns, which can be proved later through Complex Analysis methods (univalent, starlike, convex, spiral, etc.).

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ABOUT A CLASS OF LINEAR AND POSITIVE STANCU-TYPE OPERATORS

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2000 Mathematics Subject Classification. 41A36,41A60

Keywords and phrases. Bernstein polynomials; Stancu operators; King operators; fixed points.

The objective of this talk is to introduce a class of Stancu-type operators with the property that the test functions e_0 and e_1 are reproduced. Also, in our approach, two theorems of error approximation and two Voronovskaja-type theorems for this operators are obtained.

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NUMERIC EVALUATION AND CONSERVATION LAWS FOR THE DYNAMIC MODEL ASSOCIATED TO AN ACCESS CONTROL STRUCTURE

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2000 Mathematics Subject Classification. 34C14, 34B08, 34B60, 94D99

Keywords and phrases. Symmetries and invariants, parameter dependent boundary value problem, Maple assistant, fuzzy controller

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In this talk, we present some numeric evaluations regarding the dynamical behavior of an access control structure, based on the mathematical model associated to this structure.

This structure type is large analyzed in the literature. A modern approach of this structure based on SMA (shape memory alloy) is taken into account, because of some particular advantages due: unique characteristics (superelastic effect, as well as the single and double shape memory effects), damping capacity of noise and vibration, simplify and lower weight structure, resistance to fatigue (which can occur even after hundreds of thousands of cycles), diversification of the control and command possibilities.

The qualitative and numerical analysis of the mathematical model associated to this structure is taken into account. Namely, the differential equation associated to the variation of the angle θ describing the position of the access control structure is analyzed from the influence of parameters standpoint. The MAPLE soft is used in order to evaluate the behavior of the equation solution with respect to the parameters variation.

The analysis aims to find conservation laws for the dynamic model, based on the symmetries study for the ordinary differential equation. The results could be useful both for further developing a fuzzy logic controller for the active control of this access structure and for further refinements of the mathematical model associated to this structure type.

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ON THE ASYMPTOTIC BEHAVIOR OF SEQUENCES OF POSITIVE LINEAR APPROXIMATION OPERATORS

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2000 Mathematics Subject Classification. 41A10, 41A25, 41A36, 41A17, 41A80.

Keywords and phrases. Positive linear operators; central moments; rate of convergence; asymptotic scale.

We provide a sufficient condition for a sequence of positive linear approximation operators to possess a Mamedov-type property and give an application to the Durrmeyer approximation process.

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A NONLINEAR SCHRÖDINGER PROBLEM WITH MULTIPLICATIVE NOISE IN VARIATIONAL FORMULATION

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2000 Mathematics Subject Classification. 60H15, 35Q55

Keywords and phrases. Stochastic nonlinear Schrödinger equation; power-type nonlinearity; multiplicative noise; variational solution; Galerkin method

A nonlinear Schrödinger problem perturbed by multiplicative Gaussian noise will be investigated over a finite time horizon and a bounded one-dimensional domain. The appearing power-type nonlinearity has the form $f(z) = |z|^{2\sigma}z$ for $z \in \mathbb{C}$ and $\sigma \in (0, 2)$, which has many applications in mathematical physics. Being interested in the existence and uniqueness of a variational solution, a further process will be introduced which allows to transfer the stochastic Schrödinger problem into a pathwise one. Exploiting the absence of noise and using Galerkin approximations and compact embedding results, one considers first a priori estimates, existence and uniqueness of a variational solution of the pathwise Schrödinger problem. Thereafter, it is possible to extend these results to the variational solution of the nonlinear Schrödinger problem with multiplicative noise. This is a joint work with Hannelore Lisei.

AN INERTIAL FORWARD-BACKWARD ALGORITHM FOR MINIMIZING THE SUM OF TWO NON-CONVEX FUNCTIONS

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Keywords and phrases. Nonconvex optimization; Kurdyka-Lojasiewicz property; inertial forward-backward.

We introduce an inertial forward-backward algorithm for minimizing the sum of two non-convex and non-smooth functions. Building on the powerful Kurdyka-Lojasiewicz property, we derive a self-contained convergence analysis framework and establish that each bounded sequence generated by this algorithm converges.

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ON THE EVALUATION OF SOME INTEGRAL OPERATORS WITH MELLIN TYPE KERNEL

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2000 Mathematics Subject Classification. 65D30, 65D32, 65R20

Keywords and phrases. Gaussian rule; Mellin kernel; integral equations of Mellin type; Nyström method

We consider the numerical evaluation of integral transform of the form

$$(\mathcal{K}f)(y) = \int_0^1 \frac{1}{x} k\left(\frac{y}{x}\right) f(x)dx, \quad y \in (0,1], \tag{1}$$

for some given function $k : [0,\infty) \to [0,\infty)$ satisfying suitable assumptions. These operators of Mellin convolution type are not compact and their kernels are not smooth but contain a fixed strong singularity at $x = y = 0$.
The mathematical formulation of many problems in physics and engineering gives rise to the solution of second kind integral equations involving operators of the form (1). When we are interested in the numerical solution of such equations by means of Nyström or discrete collocation methods, efficient quadrature formulas are necessary, in order to approximate the integrals $(\mathcal{K}f)(y)$, $y \in (0,1]$.
The aim of this talk is to propose an algorithm for the evaluation of these integrals, since the fixed singularity of the Mellin kernel at the origin makes inefficient the use of the classical Gaussian rules when y is very close to the endpoint 0. Then, such algorithm is applied in the context of a "modified" Nyström method for the numerical solution of second kind integral equations of Mellin type. Some numerical examples illustrate the efficiency of the proposed procedures.

SOME PRESERVATIVE PROPERTIES OF A CLASS OF LINEAR POSITIVE OPERATORS

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2000 Mathematics Subject Classification. 41A36

Keywords and phrases. Linear positive operators, lipschitz constants, conservative properties

We extend some conservative properties [1] of a class of linear positive operators. In particular, we study the preservation of Lipschitz constants in several settings.

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LAGRANGE INTERPOLATION AT THE ZEROS OF POLLACZECK-TYPE POLYNOMIALS

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2000 Mathematics Subject Classification. 41A05; 41A10

Keywords and phrases. Orthogonal polynomials; Lagrange interpolation; approximation by polynomials; exponential weights.

This talk concerns the weighted polynomial approximation of functions which are continuous on the open interval $(-1, 1)$ and can increase or decrease exponentially for $|x| \to 1$. It is possible to show that a special interpolation process can realize the best polynomial approximation in suitable function spaces. To this aim we will show some estimates.

The talk is based on joint work with I. Notarangelo.

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ON THE UNBOUNDED DIVERGENCE OF INTERPOLATORY PRODUCT QUADRATURE RULES ON JACOBI NODES

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2000 Mathematics Subject Classification. 41A05

Keywords and phrases. Product quadratures; superdense unbounded divergence.

We consider interpolatory product quadrature formulas on Jacobi nodes, associated to the Banach space of all real continuous functions defined on [-1, 1] (endowed with the supremum norm), and to a Banach space of absolute integrable functions on [-1, 1], related to the L_p spaces. The main result of this talk emphasizes the phenomenon of double condensation of singularities with respect to these approximation procedures, meaning unbounded divergence on large subsets (in topological sense) of the involved Banach spaces.

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C_0-SEMIGROUP GENERATED BY A SECOND ORDER DIFFERENTIAL OPERATOR

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2000 Mathematics Subject Classification. Primary 47D06; Secondary 47D07, 47F05, 47B65

Keywords and phrases. Second order differential operator; Hilbert space; approximation process.

Let

$$Lu(x) = \frac{1}{2}x^a(1-x)^b u(x)'',$$

with $a, b \geq 2$ be a differential operator in the Hilbert space $H = L^2([0,1], \mathbb{C})$. It is shown that this operator generates a C_0 semigroup $U(t)$ in H. We study the approximation processes suitable for the semigroup together with the limiting behaviour of the semigroup for $t \to \infty$.

NEW RESULTS CONCERNING THE UNIFORM EXPONENTIAL STABILITY FOR EVOLUTION FAMILIES ON R

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2000 Mathematics Subject Classification. 34D05, 34C35, 47B48.

Keywords and phrases. Evolution family on R; exponential stability; admissibility.

The purpose of this this is to present new results for the problem of uniform exponential stability of evolution families on the real line using Perron method. We will indicate a sufficient condition for the uniform exponential stability of an evolution family on the real line that does not have exponential growth.

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THE HYERS-ULAM-RASSIAS STABILITY OF A GENERAL FUNCTIONAL EQUATION AND ITS APPLICATIONS

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2000 Mathematics Subject Classification. 26D10; 34A40; 47H10; 54H25

Keywords and phrases. Functional equation; Hyers-Ulam-Rassias stability; inequalities; gamma function; Wallis ratio

The aim of this work is to study the Hyers-Ulam-Rassias stability of a functional equation that extends the generating functional equation of the Euler gamma function. Further applications to linear recurrences, difference equations of first order, functional equations associated to Fibonacci numbers, defining functional equation of Wallis ratio are also presented.

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ON THE MONOTONICITY OF q-SCHURER-STANCU TYPE OPERATORS

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2000 Mathematics Subject Classification. 41A10

Keywords and phrases. Generalized Schurer-Stancu operators; q-integers; monotonicity; convexity.

In the last decades, the application of q-calculus represents one of the most interesting areas of research in approximation theory. We consider the class of generalized Schurer-Stancu operators in q calculus for which some properties of monotonicity and convexity are studied. Also we present numerical representations of these operators, based on Matlab algorithms.

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NONLINEAR RANDOM PARABOLIC EVOLUTION EQUATIONS IN BANACH SPACES

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2000 Mathematics Subject Classification. 37H15, 34D09, 35R60

Keywords and phrases. Random dynamical systems; stochastic differential equations; multiplicative ergodic theorem; invariant manifolds

We consider nonlinear random parabolic evolution equations given by

$$u'(t) = A(\theta_t\omega)u(t) + F(\theta_t\omega, u(t)), \;\; u(0) := x \in X.$$

Here X is a separable Banach space, A is an unbounded random operator, F is the nonlinear term and $(\theta_t)_{t\in\mathbb{R}}$ denotes a metric dynamical system. We give conditions under which such equations generate random dynamical systems and present important applications. Thereafter, we analyse the long-time behaviour of the random dynamical systems arrising in this manner. A crucial result for this purpose is a multiplicative ergodic theorem for compact operators proved in [3]. Based on this, we derive the existence of Lyapunov exponents and establish the existence of random invariant manifolds. Moreover, we provide a method of constructing the Oseledets splitting.

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NUMERICAL TREATMENT OF SOME SPECIAL FREDHOLM INTEGRAL EQUATIONS ON THE REAL SEMIAXIS

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2000 Mathematics Subject Classification. 65R20; 33C45; 41A55; 65D32

Keywords and phrases. Fredholm integral equation; weighted polynomial approximation; Nyström interpolation; Gaussian quadrature formula; orthogonal polynomials; truncation; error estimate.

The aim of this talk is to approximate the solution of integral equations of the form

$$f(x) - \mu \int_0^{+\infty} k(x,y) f(y) w(y) \, \mathrm{d}y = g(x) \,, \quad x \in (0, +\infty), \qquad (1)$$

where $\mu \in \mathbb{R}$,

$$w(y) = \mathrm{e}^{-y^{-\alpha} - y^{\beta}} \,, \quad \alpha > 0 \,, \ \beta > 1 \,,$$

the given functions k and g can grow exponentially (w.r.t. x, y) when $x \to 0^+$ and/or $x \to +\infty$.

The methods based on the weighted polynomial approximation with Laguerre-type weights are not suitable in this case and only recently, the authors have studied the polynomial approximation with the weight w (see [1]) and the behavior of the related Gaussian rules, considering also the computation of the zeros and the Christoffel numbers (see [2]).

So, for the numerical treatment of equation (1), we are going to use a Nyström method based on a suitable "truncated" Gaussian rule. We prove the stability and the convergence of the method and give a priori estimates of the error.

The talk is based on joint work with G. Mastroianni and G.V. Milovanović.

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LAGRANGE INTERPOLATION ON UNBOUNDED INTERVALS AND SOME APPLICATIONS

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In this talk it is proposed a new Lagrange interpolation process based on some knots $[\{\zeta_k\}_{k=1}^m]_{m\in N}$ related to the Laguerre zeros $[\{x_{m,k}(w_\alpha)\}_{k=1}^m]_{m\in N}$, $w_\alpha(x) = e^{-x}x^\alpha$.

As an application it will be shown how to approximate the Hilbert transform

$$\int_0^{+\infty} \frac{f(x)}{x-t} w_\alpha(x)dx, \tag{1}$$

by means of the previous Lagrange interpolation combined with the Truncated Gauss-Laguerre rule [1]. This procedure improves in some sense that introduced in [2], where two different Laguerre polynomial sequences are employed.

Finally, some numerical experiments will be shown to evidence the behavior of the Lebesgue functions and some examples will be proposed to test the efficiency in approximating the Hilbert transform (1).

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KOROVKIN THEOREM FOR QUASI-POSITIVE SEQUENCES OF OPERATORS

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2000 Mathematics Subject Classification. 41A35, 41A50

Keywords and phrases. Monotone operators; sequence of quasi-positive operators; interpolation; best approximation.

We consider sufficient and necessary conditions for a sequence of quasi-positive sequence of monotone operators in order to have the property of uniform approximation of continuous functions.

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FULLY DEVELOPED MIXED CONVECTION THROUGH A VERTICAL POROUS CHANNEL WITH AN ANISOTROPIC PERMEABILITY: CASE OF HEAT FLUX

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2000 Mathematics Subject Classification. 75R99, 68W30

Keywords and phrases. Mixed convection; heat flux; symbolic computation; computer algebra.

The effect of anisotropy on the steady fully developed mixed convection flow in a vertical porous channel is analytically studied. The side walls of the channel are prescribed by a constant heat flux and the flow at the entrance is upward, so that natural convection aids the forced flow. It is shown that the anisotropy parameter has a significant effect of the flow and heat transfer characteristics. This work is an extension of [1].

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OPTIMAL CUBIC LAGRANGE INTERPOLATION: EXTREMAL NODE SYSTEMS WITH MINIMAL LEBESGUE CONSTANT

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2000 Mathematics Subject Classification. 05C35, 33F10, 41A05, 41A44, 68W30, 97N50.

Keywords and phrases. Cubic; extremal; interpolation; Lagrange interpolation; Lebesgue constant; minimal; node system; optimal; polynomial; symbolic computation.

In the theory of interpolation of continuous functions by algebraic polynomials of degree at most $n \geq 2$, the search for an analytic expression of extremal (or: optimal) node systems which minimize the Lebesgue constant is still an intriguing topic in mathematics today (see, e.g., [1], [2], [4, p. 67], [11]), and according to [3, p. xlvii] the nature of the optimal nodes remains a mystery. However, the quadratic case $n = 2$ has been completely resolved (see [7], [8]), and in this talk we are going to resolve the cubic case $n = 3$. To this end, we consider optimal cubic Lagrange interpolation which asks to determine (among all possible node systems $\Delta_4 : -1 \leq x_1 < x_2 < x_3 < x_4 \leq 1$) all extremal node systems $\Delta_4^* : -1 \leq x_1^* < x_2^* < x_3^* < x_4^* \leq 1$ that lead to the minimal cubic Lebesgue constant Λ_4^* whose numerical value is $1.42291\ldots$. An explicit analytic expression for Λ_4^* and for the extremal zero-symmetric canonical node system $-1 = -x_4^* < -x_3^* < x_3^* < x_4^* = 1$ has already been provided by the first author [5],

[6]. An implicit expression for Λ_4^* and for all extremal zero-symmetric node systems $-1 \le -x_4^* < -x_3^* < x_3^* < x_4^* \le 1$ (which encompass the extremal canonical one) had already been provided some fifty years ago by Tureckii [9], [10]. Here we provide explicit descriptions, which seem to be novel, of all the infinitely many extremal (zero-symmetric and zero-asymmetric) node systems Δ_4^*. Our solution of the cubic case of optimal polynomial Lagrange interpolation is guided by symbolic computation.

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STANCU OPERATORS - A SURVEY OF ITS LINEAR COMBINATIONS AND GENERALIZATIONS

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2000 Mathematics Subject Classification. 41A10, 41A36

Keywords and phrases. Approximation by positive linear operators; Stancu operators.

More than forty years ago, in 1968 in paper [2], D.D. Stancu introduced and studied a new sequence of linear and positive operators, $S_n^\alpha : C[0,1] \to C[0,1]$,

$$\left(S_n^\alpha f\right)(x) = \sum_{k=0}^{n} \omega_{(n,k)}^{\alpha}(x) f\left(\tfrac{k}{n}\right)$$

where

$$\omega_{(n,k)}^{\alpha}(x) = \binom{n}{k} \frac{x^{[k,-\alpha]}(1-x)^{[n-k,-\alpha]}}{1^{[n,-\alpha]}},$$

$n \in \mathbb{N}$ and α beeing a real parameter depending only on n. We remind that $\omega_{(n,k)}^{\alpha}$ are known as "the fundamental polynomials of Stancu". This talk is concerned with linear combinations of the Stancu polynomials. The idea was inspired by [1]. In this talk we describe other generalizations and we summarize various results due to several mathematicians who have studied Stancu operators.

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SPECIAL FUNCTIONS ASSOCIATED WITH POSITIVE LINEAR OPERATORS

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2000 Mathematics Subject Classification. 33C05, 42C05, 41A36

Keywords and phrases. Special functions; orthogonal polynomials; positive linear operators.

We consider the sums of the squared fundamental functions of some classical positive linear operators. They can be expressed in terms of special functions and orthogonal polynomials. Consequently, we show that they are solutions to some classical differential equations: in particular, Heun type equations.

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APPROXIMATING SCHEMES FOR BSVIs WITH GENERALIZED REFLECTION

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2000 Mathematics Subject Classification. 65C99, 60H30, 47H15

Keywords and phrases. Euler-Yosida approximation; BSVIs.

Abstract. We define approximation schemes for generalized backward stochastic differential systems, considered in the Markovian framework. More precisely, we propose a mixed approximation scheme for the following backward stochastic variational inequality

$$dY_t + F(t, X_t, Y_t, Z_t)dt \in \partial\varphi(Y_t)dt + Z_t dW_t,$$

where $\partial\varphi$ is the subdifferential operator of a convex lower semicontinuous function φ and $(X_t)_{t\in[0,T]}$ is the unique solution of a forward stochastic differential equation. We use an Euler type scheme for the system of decoupled forward-backward variational inequality joint with Yosida approximation techniques. We also present an approximating scheme for (Markovian) BSVIs with oblique reflecting subgradients.

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ROTATION NUMBERS FOR RANDOM DYNAMICS ON THE CIRCLE

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2000 Mathematics Subject Classification. 37C40, 37E45, 37E10, 37A05

Keywords and phrases. Rotation number; discrete random dynamics; homeomorphisms of the circle.

We revisit the problem of well-defining rotation numbers for discrete random dynamical systems on S^1. We show that, contrasting with deterministic systems, the topological (i.e. based on Poincaré lifts) approach does depend on the choice of lifts (e.g. continuously for nonatomic randomness). Furthermore, the winding orbit rotation number does not agree with the topological rotation number. Existence and conversion formulae between these distinct numbers are presented. Finally, we prove a sampling in time theorem which recover the rotation number of continuous Stratonovich stochastic dynamical systems on S^1 out of its time discretisation of the flow.

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APPROXIMATION BY STANCU-KANTOROVICH OPERATORS BASED ON q-INTEGERS

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2000 Mathematics Subject Classification. 41A10, 41A25, 41A36.

Keywords and phrases. q-Stancu-Kantorovich operators; modulus of continuity; rate of convergence; Voronovskaja theorem.

The goal of our talk is to introduce new q-Stancu-Kantorovich operators and to study some of their approximation properties. A convergence theorem using the well known Bohman-Korovkin criterion is proven and the rate of convergence involving the modulus of continuity is established. Furthermore, a Voronovskaja type theorem is also proven.

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B-SPLINE FRACTAL INTERPOLATION

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2000 Mathematics Subject Classification. 65D05

Keywords and phrases. Fractals; interpolation.

The classical methods of real data interpolation can be generalized with fractal interpolation. These fractal interpolation functions provide new methods of approximation of exprimental data. We extend the B-spline interpolation method by means of a class of fractal interpolants.

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AN OPTIMAL SET OF QUADRATURE RULES FOR TRIGONOMETRIC POLYNOMIALS IN THE SENSE OF BORGES

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2000 Mathematics Subject Classification. 65D32; 42C05

Keywords and phrases. Multiple orthogonality; recurrence relations; optimal set of quadrature rules

In this talk we consider an optimal set of quadrature rules in the sense of Borges [Numer. Math. **67** (1994), 271–288] with an odd number of nodes for trigonometric polynomials. As a matter of fact we consider evaluation of a set of $p \in \mathbb{N}$ definite integrals related to a common integrand over the same interval E of length 2π, but taken with respect to the different weight functions. The optimal set of quadrature rules is characterized by multiple orthogonal trigonometric polynomials of semi–integer degree. We give main properties of such multiple orthogonal system as well as the numerical procedure for constructing the corresponding quadrature rules. Theoretical results are illustrated by some numerical examples.

ON GLOBAL SMOOTHNESS PRESERVATION BY BERNSTEIN-TYPE OPERATORS

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2000 Mathematics Subject Classification. 41A36, 41A17.

Keywords and phrases. Global smoothness preservation; second order modulus of continuity; Bernstein-type operators.

We study global smoothness preservation of a function f by sequences of Bernstein-type operators with respect to some moduli of continuity of order two.

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ON JAIN LINEAR OPERATOR AND ITS EXTENSIONS

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2000 Mathematics Subject Classification. 41A36

Keywords and phrases. Linear positive operator; Kantorovich-type operator; Durrmeyer-type operator; q-Calculus.

By using the Poisson-type distribution given by

$$w_\beta(k;\alpha) = \tfrac{\alpha}{k!}(\alpha + k\beta)^{k-1}e^{-(\alpha+k\beta)},\ k \in \mathbb{N}_0,$$

for $0 < \alpha < \infty$ and $|\beta| < 1$, G.C. Jain [1] introduced and studied the following class of positive linear operators

$$(P_n^{[\beta]}f)(x) = \sum_{k=0}^{\infty} w_\beta(k;nx) f\left(\tfrac{k}{n}\right),\ x \geq 0,$$

where $\beta \in [0,1)$ and $f \in C(\mathbb{R}_+)$.

The first goal of our talk is to investigate in more detail this class, revealing new properties of its. Further on we present a q-generalization of this family and two integral extensions in Durrmeyer sense and respectively, in Kantorovich sense. Approximation properties of these new classes are highlighted.

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SEMI-ANALYTICAL SOLUTION FOR A CLASS OF BOUNDARY VALUE PROBLEMS

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2000 Mathematics Subject Classification. 34B15, 68W30, 65L10

Keywords and phrases. Symbolic computation; computer algebra; nonlinear equations.

We try to solve a boundary value problem from fluid mechanics using a computer algebra software (Maple) in two phases. If we do not take into account the boundary conditions, we obtain an analytic solution in terms of hypergeometric functions and several free constants. In a second phase we obtain the constant by solving a nonlinear system.

ON CONVERGENCE OF A KIND OF COMPLEX NONLINEAR BERNSTEIN OPERATOR

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2000 Mathematics Subject Classification. 41A35, 41A25, 47G10

Keywords and phrases. Nonlinear Bernstein operators; Lipschitz condition; Voronovskaya-type result; compact disks.

The aim of our talk is to present the approximation properties and Voronovskaya type results with quantitative estimates for a certain class of complex nonlinear Bernstein operator of the form

$$(NB_n f)(z) = \sum_{k=0}^{n} P_{k,n}\left(z, f\left(\tfrac{k}{n}\right)\right), \qquad |z| < 1,$$

attached to analytic functions on compact disks.

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NUMERICAL DETECTION OF DETERMINISTIC COMPONENTS OF NOISY ASTRONOMICAL TIME SERIES

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2000 Mathematics Subject Classification. 62M10, 85A35

Keywords and phrases. Self-correlation function; detection of deterministic signals.

The usual method for periodicity detection in astronomical noisy time series relies on the amplitude spectrum analysis and the statistical significance of the highest peak can be estimated through Monte Carlo simulations. A complementary numerical method is the self-correlation analysis [1, 2, 3]. An important deficiency of this method is the lack of information concerning the statistical properties of the self-correlation function [4]. We study these properties analytically and through Monte Carlo simulations. We also apply it to a pathological data set describing the orbital period variability of the eclipsing binary system ER Vulpeculae. This time series contains only 91 timing data, is unevenly sampled, is dominated by noise, and is interrupted by two large time gaps. Using the self-correlation method we succeeded to improve the confidence level for the rejection of the null hypothesis that there is no deterministic component in the observed time series from 96% to 99.9% [5].

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OUR BELOVED MASTER DIMITRIE D. STANCU
In memoriam

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2000 Mathematics Subject Classification. 01A70

Keywords and phrases. D.D. Stancu; The Romanian School of Numerical Analysis and Approximation Theory.

We emphasize some essential aspects of the life and the work of our master Acad. prof. Dr. H. C. Dimitrie D. Stancu.

CONVERGENCE ANALYSIS OF A STRANG SPLITTING METHOD FOR TIME-DEPENDENT KOHN-SHAM EQUATIONS

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2000 Mathematics Subject Classification. 65M1

Keywords and phrases. Strang splitting; coupled Schrödinger equations; error analysis.

The time-dependent Kohn-Sham equations

$$\begin{aligned} & i\frac{\partial}{\partial t}u_1 = -\Delta u_1 + V u_1 \\ & \dots \\ & i\frac{\partial}{\partial t}u_n = -\Delta u_n + V u_n \\ & -\Delta V = \sum_{k=1}^{n} |u_k|^2 \end{aligned}$$

form a system of coupled Schrödinger equations which is of great interest in physical chemistry, especially in molecular dynamics simulations.

For this system, we study the approximation properties of a semi-discretization (in time), namely a Strang-type splitting method, which is a composition of the exact flows generated by the linear part and the nonlinear part of the system.

Although such methods are widely used, to our knowledge, there is as yet no rigorous convergence result in literature concerning the time-dependent Kohn-Sham equations.

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ON CONVERGENCE OF NONLINEAR SINGULAR INTEGRAL OPERATORS WITH NON-ISOTROPIC KERNELS

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Keywords and phrases. Nonlinear singular integral; non-isotropic distance; Lipschitz condition.

Here we give some approximation theorems concerning pointwise convergence and rate of pointwise convergence of nonlinear singular integral operators with non-isotropic kernels of the form:

$$T_{w,\lambda}(f)(s) = \int\limits_{\mathbb{R}^n} K_w \left(|s-t|_\lambda , f(t) \right) dt,$$

satisfying Lipschitz and some singularity assumptions. Here Λ is a non-empty set of indices, w_0 is an accumulation point of Λ and $|s-t|_\lambda$ denotes the non-isotropic distance between the points $s, t \in \mathbb{R}^n$.

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ON SOME STOCHASTIC OPTIMAL PROBLEMS FOR AN ENERGY STORAGE FACILITY

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Keywords and phrases. Stochastic optimal control; HJB equation; energy markets

We address a real option problem consisting of the valuation of a energy storage facility in the presence of stochastic energy prices. Such problems arise in case of natural gas dome storage and hydroelectric pumped storage. The valuation problem is related to the problem of determining the optimal injection/withdrawal strategy that maximizes the expected value of the resulting discounted cash flows over the lifetime of the storage. Special features are operational constraints on the strategy and the storage level.

We formulate the problem as a stochastic control problem in continuous time, resulting in a Hamilton-Jacobi-Bellman (HJB) equation. We use numerical methods such as policy improvement, finite difference schemes and a semi-Lagrangian technique for solving the HJB equation. Our approach is able to handle a wide class of energy price models that exhibit mean-reverting, seasonality dynamics and dependence on unobservable factor processes. Finally, we illustrate our results numerically.

The talk is based on joint work with Anton Shardin.

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